

# Optimizing Point to Point Motion of Net Velocity Constrained Manipulators

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**Abstract**—The architecture of many hydraulic manipulators, such as excavators common in the earthmoving industry, have constraints on the net sum of actuator speeds. This paper gives the necessary conditions for minimum-time velocity commands for point to point motion. A kinematic model of the manipulator is used. The optimal solution is not always unique. We propose a particular optimal solution,  $u^*$ , that is stationary. The optimality of inputs unequal to  $u^*$  is evaluated by the position of  $u^*$  in the input domain. Several examples are given to demonstrate the analysis.

## NOMENCLATURE

$C$	Maximum pump flow rate.
$D$	Vector of single-actuator flow constraints.
$\psi$	Vector of the flow-velocity ratio of each actuator.
$u$	Velocity of actuators.
$u^*$	A useful point for testing optimality.
$x$	Normalized position coordinate.
$q$	Generalized position coordinate.
$(\cdot)^T$	Transpose of $(\cdot)$ .
$(\cdot)_k$	The $k$ th element of vector $(\cdot)$ .

## I. INTRODUCTION

Hydraulic actuators are used in applications requiring high power density at low to moderate speeds, including large-scale industrial manipulators for factory automation and earthmoving. Here, we consider the common earthmoving excavator in Fig. 1. Excavators generally have at least four degrees of freedom arranged in an open kinematic chain and are typically manually controlled by a human operator seated in the cab. That excavators are ubiquitous, multi-DOF mobile manipulators capable of performing a wide array of functions makes them excellent testbeds for studying robotics and controls.

All manipulators are subject to motion constraints including power limits and joint torque limits. In addition, multi-DOF hydraulic manipulators driven by a single source of pressurized oil are subject to limits on the combined velocity achievable by all actuators. This constraint does not generally affect electrically actuated robots, and consequently the literature is deficient in highlighting methods of control to deal with this problem.

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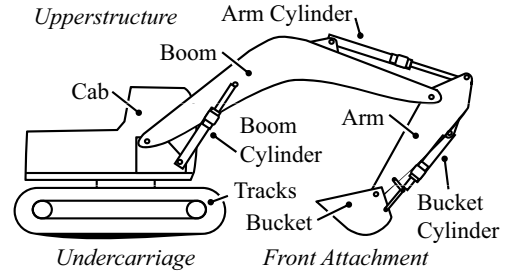


Fig. 1: Major components of an excavator; the four actuated functions are swing, boom, arm, bucket.

We have recently proposed a technique termed *blended shared control* (SC) as a way to decrease task time of manually controlled systems [1]. Blended SC may be a low overhead way to decrease cycle times of repetitive, manually controlled tasks. A key module of blended SC is a method of calculating the time-optimal control to move the manipulator from one configuration to another. The optimization must be completed in real-time and for a variety of configurations. Optimization methods presented in literature are too specialized [2] or appear to be unsuitable for real-time implementation [3], [4], [5]. This paper starts from the assumption of quasi-static dynamics. From here, necessary optimality conditions on the input  $u$  are stated. A special point,  $u^*$ , is introduced. In the case where the input  $u(t) = u^*$  for all time, the motion is necessarily optimal. In the case where  $u(t) \neq u^*$  for some time, the input may or may not be optimal. The location of  $u^*$  in the domain of allowable inputs  $U$  provides a convenient test for optimality, and can discriminate an input  $u(t)$  as sub-optimal even before the input violates a necessary condition for optimality.

## II. SYSTEM BACKGROUND

The manipulator linkages are accelerated by a hydraulic actuator system comprised of cylinders, conduits, controlled orifices, pumps, accumulators, and a prime mover. Fig. 2 shows a simplified schematic of the typical connection actuators for the type of systems considered here. The valves represent a generic arrangement of electronically controlled orifices. The separate orifices comprising each valve may be independently controlled (e.g., the valve may consist of four or more electro-hydraulic poppet valves) or they may be coupled (e.g., the valve may consist of a spool valve with a single degree of freedom). An open-loop, electronic valve controller handles low-level actuator tracking control.

Many such manipulators have a single pump to supply

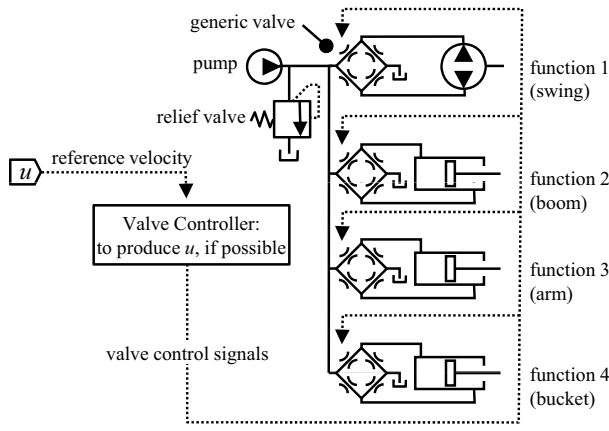


Fig. 2: The circuit for a multi-DOF hydraulic manipulator. Supporting hydraulic circuitry and components are not shown.

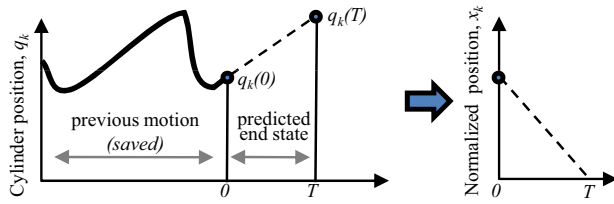


Fig. 3: Piecewise monotonic motion segments are normalized so the value at the final time is zero.

flow to all actuators as in Fig. 2. The pump is often undersized so it is impossible to simultaneously actuate all functions at full speed.

### III. THE MANIPULATOR TASK

Let  $q(t) = [q_1(t), \dots, q_n(t)]^T$  be the generalized position of the actuators, e.g. the cab rotation (or *swing*) angle is  $q_1$ , and the length of the boom, arm, and bucket cylinders are  $q_2, q_3, q_4$ , respectively. In absence of kinematic singularities, any end effector path through the workspace is equivalently described by the displacement  $q(t)$  of the actuators. An actuator trajectory may be decomposed into a sequence of piecewise monotonic segments termed *motion primitives*. We assume that the motion between the endpoints of these motion primitives is inconsequential, and that only the final relative displacement is to be considered.

A change of variables simplifies the notation. Defining

$$x(t) = (q(T) - q(t)) \text{sign}(q(T) - q(0)) \quad (1)$$

as illustrated in Fig. 3 makes the motion from some starting point  $q(t)$  to the end of the current motion primitive occurring at  $q(T)$  equivalent to the motion from  $x(t)$  to  $x(T) = 0$ . The  $\text{sign}(\cdot)$  term guarantees that  $x(t) \geq 0$ , since  $q(t)$  is monotonic over the primitive. With the change of variables, the problem is to drive the  $x(t)$  system from an initial position  $x(0)$  to the origin,  $x(T) = 0$ , with minimum

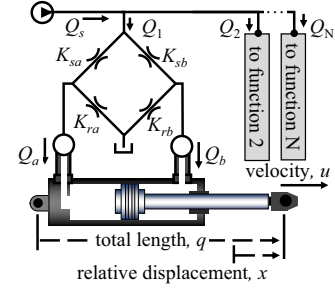


Fig. 4: A cylinder and generic hydraulic valve with controlled orifices.

final time  $T$ . Note that  $x(t)$  denotes the expected remaining actuator displacement before reaching the origin.

#### A. System Model

Excavators have dynamics that occur over very different time scales, ranging from very fast pressure rise within a closed volume of fluid to slower rigid-body linkage dynamics [6]. Fluid power researchers studying the gross motion of hydraulic manipulators often assume the hydraulic components undergo “quasi-static” dynamics.

We also make this convenient, though not entirely accurate, assumption that all the hydraulic dynamics can be neglected. Thus, assuming the actuators follow a simple kinematic velocity-controlled law gives

$$\dot{x} = -u \quad (2)$$

with the assumption that

$$u \in U$$

where the velocity control input to the cylinders is  $u$  and  $U$  is the set of allowable inputs, as defined in Sec. IV. The piecewise optimization of kinematic manipulators has been treated previously [7]; however, the constraints considered here place limits on the sum of actuator velocities—a constraint class not typically covered in literature.

### IV. LIMITS ON CYLINDER VELOCITIES

Here we show how the space  $U$  of allowable velocities is related to the flow-velocity ratio (FVR)  $\psi$  of each actuator.

#### A. Flow-velocity Ratio

Flow control valves direct pressurized oil from the system pump to the actuators, as illustrated in Fig. 4. The flow entering the actuator is related to the valve’s operating mode, i.e., the particular combination of open orifices, which also determines the FVR of that actuator. The FVR relates the steady-state velocity  $u_k$  of the actuator to  $Q_k$ , the portion of the system pump flow  $Q_s$  that enters function  $k$ , as

$$\psi_k \equiv \frac{Q_k}{u_k} \quad (3)$$

By defining relative displacements as in (1), the velocity  $u_k$  is always nonnegative. The flow  $Q_k$  entering the actuator control valve from the supply conduit may be zero (if the valve is operating in a *regeneration* mode) or positive. Thus  $\psi_k \geq 0$ .

### B. Multi-actuator Velocity Constraint

Let  $C > 0$  denote the maximum flow rate of the pump. The combined velocity of all actuators is limited and must satisfy

$$\sum Q_k = \sum \psi_k u_k = \psi^T u \leq C \quad (4)$$

where  $\psi = [\psi_1, \dots, \psi_n]^T$  and  $u = [u_1, \dots, u_n]^T$ . We assume that the valve operating mode, and hence  $\psi$ , is constant throughout the motion from  $x(t)$  to the origin.

### C. Single-actuator Velocity Constraint

The maximum actuator speed is constrained by the max flow that can pass through the valve. The velocity of actuator  $k$  must satisfy

$$u_k \leq D_k \quad (5)$$

where the parameter  $D_k$  depends on the valve operating mode, the orifice physical limitations, the maximum supply pressure, and the power limits of the system.

### D. Domain of Allowable Control Inputs

The allowable control inputs  $u$  must satisfy (4) and (5); thus the domain of feasible velocities  $u$  is

$$U = \{u : 0 \leq u_k \leq D_k, \psi^T u \leq C\} \quad (6)$$

The region  $U$  is illustrated as the shaded region in Fig. 5 for a two-dimensional case.<sup>1</sup>

### E. Projection of Non-allowable Inputs to Allowable Region

The human operator commands the actuator velocity  $\bar{u}$  by displacing joysticks located within the excavator cab. From the equations of motion (2), the actuators track the input  $u$  perfectly for  $u \in U$ . If the velocity  $\bar{u}$  commanded by the operator is outside  $U$ , then  $\bar{u}$  is first projected to the feasible region. A common technique in industry [8] is to scale each component of  $\bar{u}$  by a common factor  $\alpha$  so that

$$\bar{u}_p = \text{proj}_{\partial U} \bar{u} \quad (7)$$

$$= \alpha \bar{u} \quad (8)$$

where  $\bar{u}_p$  is the projection of  $\bar{u}$  onto the boundary  $\partial U$  of  $U$ . With reference to Fig. 5,  $\partial U$  may consist of points along the multi-actuator constraint  $\psi^T u = C$ , or the single-actuator constraints  $u_k = D_k$ , or both. The scalar  $\alpha$  is

$$\alpha = \min_j (1, \alpha_m, \alpha_j) \quad (9)$$

where  $\min_j (\cdot)$  indicates the minimum of the arguments considering all values  $j = 1, 2, \dots, n$  and

$$\alpha_m = \frac{C}{\psi^T \bar{u}} \quad (10)$$

$$\alpha_j = \frac{D_j}{\bar{u}_j} \quad (11)$$

are the factors necessary to scale  $\bar{u}$  to intersect the multi- and single-actuator constraint lines.

<sup>1</sup>All figures will be drawn in the  $u_1$ - $u_2$  plane for clarity; however, in general,  $u \in R^n$ .

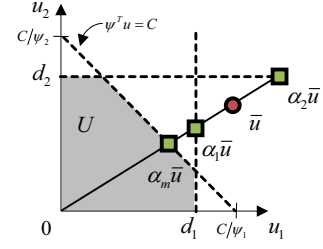


Fig. 5: An infeasible input  $\bar{u}$  is projected into the feasible region  $U$  by proportionally scaling each component of  $\bar{u}$ .

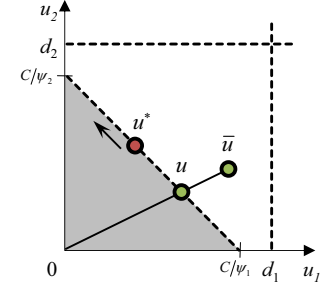


Fig. 6: Allowable region  $U$  for an undersized pump with  $D_k \geq C \forall k$ . An infeasible  $\bar{u}$  is projected to the boundary of  $U$  by scaling each component equally.

Formally, the problem is summarized as

$$\begin{aligned} &\text{Find } u(t) \text{ to minimize } T \\ &\text{subject to } x(0) = x_0 \\ &\quad x(T) = 0 \\ &\quad \dot{x} = -u \\ &\quad u \in U \end{aligned} \quad (12)$$

Problems similar to 12 have been posed and solved in various ways including dynamic programming, optimal control [9], and by inversion of the main system dynamics [10] of a chemical process.

### V. OPTIMAL INPUT - SPECIAL CASE OF UNDERSIZED PUMP

If  $D_k \geq C \forall k$ , then the pump is *undersized*, and the feasible region in the  $u$ -plane is triangular as sketched in Fig. 6. The optimal solution to (12) will satisfy

$$\psi^T u(t) = C \quad (13)$$

for all time.

There is a particular  $u$ , denoted  $u^*$ , for which all actuators reach the origin simultaneously. This input is constant for all time throughout the trajectory from  $x(0)$  to the origin.

*Theorem 1:* The input

$$u^* = \frac{C}{\psi^T x} x \quad (14)$$

will drive the system (12) from  $x = [x_1, \dots, x_n]^T$  to the origin in minimum time with a final time

$$T^* = \frac{\psi^T x}{C}$$

and will be constant for all  $x$  along the trajectory.

*Proof:* The total fluid volume required from the system pump is related to the flow-velocity ratio  $\psi$  of each actuator and the remaining distance  $x$  each actuator must travel as

$$V = \psi_1 x_1 + \psi_2 x_2 + \dots + \psi_n x_n = \psi^T x$$

With input  $u^*$  the pump always delivers maximal flow rate  $C$  because  $\psi^T u^* = C \psi^T x / (\psi^T x) = C$ . Hence, the minimum time to deliver the net volume  $V$  is

$$T^* = \frac{\psi^T x}{C}$$

Given a constant input  $u_k$ , the time required for the  $k$ th actuator to go from  $x_k$  to the origin is

$$T_k = \frac{x_k}{u_k}$$

Equating  $T_k = T^* = \frac{\psi^T x}{C}$  and solving and solving for  $u_k$ , gives

$$u_k^* = \frac{C}{\psi^T x} x_k$$

for all  $k = 1, \dots, n$ , which becomes (14) when expressed in vector notation. ■

#### A. Motion of $u^*$ for Suboptimal $\bar{u}$

If the manipulator is manually controlled, and the projected operator input  $\bar{u}_p$  differs from  $u^*$ , then  $u^*$  will in general not be stationary. Understanding the dynamics of  $u^*$  is helpful for assessing the optimality of an input. The motion of  $u^*$  will depend on the state  $x$  and the input  $u$ , as in Theorem 2.

*Theorem 2:* The point  $u^*$  defined by (14) is a dynamic function of the state  $x$  and input  $u$ , having velocity  $du^*/dt = \dot{u}^*$ , where

$$\dot{u}^* = \frac{C}{(\psi^T x)^2} ((\psi^T u) x - (\psi^T x) u) \quad (15)$$

*Proof:* This result follows directly from

$$\frac{du^*}{dt} = \nabla u^* \frac{dx}{dt}$$

by using the dynamics in (2) and

$$\nabla u^* = \frac{C}{\psi^T x} [I] - \frac{C}{(\psi^T x)^2} x \psi^T$$

where  $[I]$  is the identity matrix. The velocity is always directed parallel to the manifold  $\psi^T u = C$  because

$$\psi^T \dot{u}^* = \frac{C}{(\psi^T x)^2} (\psi^T (\psi^T u) x - \psi^T (\psi^T x) u) \equiv 0$$

■

*Remark 1:* The point  $u^*$  is stationary for any  $u$  along the line from the origin to  $u^*$ . This is true because  $\dot{u}^* = 0$  whenever  $u_k/x_k = (\psi^T u) / (\psi^T x)$ .

*Remark 2:* The velocity  $\dot{u}^*$  is always on the constraint manifold  $\psi^T u = C$  and points in a direction “away” from  $u$ . This result is seen by writing (15) as

$$\begin{aligned} \dot{u}^* &= \frac{\psi^T u}{\psi^T x} \frac{C}{\psi^T x} x - \frac{C}{\psi^T x} u \\ &= \frac{\psi^T u}{\psi^T x} u^* - \frac{C}{\psi^T x} u \\ &= \frac{\psi^T u}{\psi^T x} (u^* - u) - \frac{C - \psi^T u}{\psi^T x} u \end{aligned}$$

Thus, there is always a component of velocity in the direction  $(u^* - u)$ . The motion of  $u^*$  for the case of suboptimal operator input ( $u = \bar{u}$ ) is illustrated in Fig. 6; for the planar case,  $u^*$  moves along the line  $\psi^T u = C$ .

## VI. OPTIMAL INPUT FOR GENERAL CASE

With input  $u = \bar{u}_p \in U$ , the task time  $T$  is the time required for all components  $x_k$  to reach the origin, and can be written as

$$T = \max_i \frac{x_i}{u_i}$$

Choosing  $u = u_p^*$ , where  $u_p^*$  is the the projection of  $u^* = \frac{C}{\psi^T x} x$  onto the feasible region using (8), gives

$$u_p^* = \min_j \left( 1, \frac{C}{\psi^T u^*}, \frac{D_j}{u_j^*} \right) u^*$$

so that

$$T^* = \max_i \left( \frac{\frac{x_i}{u_i^*}}{\min_j \left( 1, \frac{C}{\psi^T u^*}, \frac{D_j}{u_j^*} \right)} \right)$$

where

$$\begin{aligned} \frac{C}{\psi^T u^*} &= \frac{C}{\psi^T x} \frac{\psi^T x}{C} = 1 \\ \frac{D_j}{u_j^*} &= \frac{D_j}{\frac{C}{\psi^T x} x_j} = \frac{\psi^T x}{C} \frac{D_j}{x_j} \\ \frac{x_i}{u_i^*} &= \frac{x_i}{\frac{C}{\psi^T x} x_i} = \frac{\psi^T x}{C} \end{aligned}$$

Thus the optimal completion time can be written as

$$T^* = \max_i \left( \frac{\psi^T x / C}{\min_j \left( 1, \frac{\psi^T x}{C} \frac{D_j}{x_j} \right)} \right) \quad (16)$$

$$= \left( \frac{\psi^T x / C}{\min_j \left( 1, \frac{\psi^T x}{C} \frac{D_j}{x_j} \right)} \right) \quad (17)$$

$$= \max_j \left( \frac{\psi^T x}{C}, \frac{x_j}{D_j} \right) \quad (18)$$

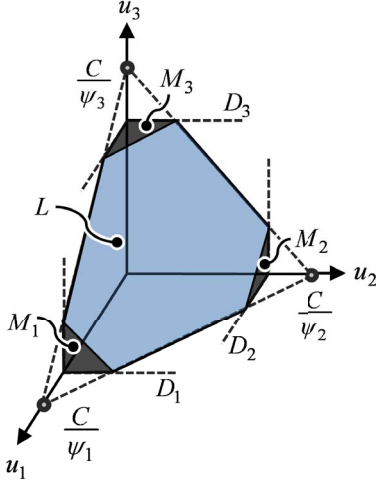


Fig. 7: The regions  $L$  and  $M_k$  in the domain  $U$  for  $n = 3$

## VII. OPTIMALITY CONDITIONS

There are two subsets of the boundary of  $U$ ,  $\partial U$ , which are of interest. Let  $L$  and  $M_k$  be the regions defined as

$$L = \{u : \psi^T u = C, \quad u_k \leq D_k\} \quad (19)$$

$$M_k = \{u : u_k = D_k, \quad u \in U\} \quad (20)$$

$u \in L$  requires the maximum pump flow  $C$ , while  $u \in M_k$  implies that  $u_k$  is at maximum value,  $D_k$ , for actuator  $k$ . Fig. 7 shows a sketch of regions  $L$  and  $M_k$  for the case of three actuators ( $n = 3$ ). Note, it is not necessary for  $M_i \cup M_j = \emptyset$ .

**Claim 1:** If  $u^* \in L$ , then the minimum task time is  $T^* = \frac{\psi^T x}{C}$ . The optimality conditions are

$$1) \quad u(t) \in L \quad \forall t \in [t_0, t_0 + T]$$

$$2) \quad u^*(t) \in L \quad \forall t \in [t_0, t_0 + T]$$

Note that (1)  $\Rightarrow$  (2).

**Claim 2:** If the projected point  $u_p^* \in M_k$ , then a necessary condition for optimality is that  $u_p^*(t) \in M_k \quad \forall t \in [t_0, t_0 + T]$ .

While not shown here for succinctness, Claim 1 and Claim 2 are proven by showing that the task time when satisfying these claims equals the optimal time given in (18).

## VIII. EXAMPLES

**Example 1, in Fig. 8:** Consider manipulation of two actuators, with  $\psi = [1, 1]^T$ ,  $C = 2$ , and  $d_k > 2$ . Suppose the actuators are just beginning a motion with  $x(0) = [1, 1]^T$ . Using (14), the stationary optimal input is  $u^* = [1, 1]^T$ . Suppose the operator input is  $\bar{u} = [\frac{1}{4}, 1\frac{3}{4}]^T$ , then the point  $u^*$  moves away from  $\bar{u}$  in the direction shown in Fig. 8a.

The operator input  $\bar{u}$  satisfies the necessary condition for optimality ( $\psi^T \bar{u} = C$ ) up to the moment that  $u^*$  enters the  $u_2 = 0$  plane. At this time, the dimensionality of the problem is reduced to a line as in Fig. 8b. The original input  $\bar{u}$  violates the optimality condition since  $\psi_2 \bar{u}_2 < C$ . To remain optimal, the operator input must always lie within the locus of point  $\psi^T u = C$ . Hence, if the operator does not immediately

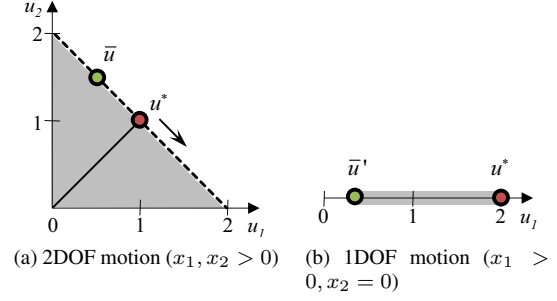


Fig. 8: Example 1: Example of dynamics within  $u$ -plane for 2 degree-of-freedom manipulation

change inputs when the dimensionality is reduced, then there will be a time for which the input is not optimal.

Consider three hypothetical trajectories for a 2DOF manipulator moving from the initial state  $x = [1, 1]^T$  to the origin. The illustrations will assume  $C = 1$ ,  $\psi = [1, 1]^T$ , and  $D = \frac{3C}{4}\psi$ .

**Example 2, in Fig. 9:** The input is chosen to be  $\bar{u} = u^*$ , so the necessary condition for optimality ( $\psi^T u = C$ ) is satisfied everywhere. In the  $u$ -plane, Fig. 9b, the point  $u^*$  is stationary.

**Example 3, in Fig. 10:** Here,  $\bar{u} \neq u^*$ ; however, the necessary condition for optimality is satisfied ( $\psi^T \bar{u} = C$  always). Consider the behavior of  $u^*$  in the  $u$ -plane. Initially,  $\bar{u} = [\frac{1}{4}, \frac{3}{4}]^T$  as shown in Fig. 10b. The point  $u^*$  has a velocity away from  $\bar{u}$ , so it slides down the line  $\psi^T u = C$ . Just before  $t = 1$ , the point  $u^*$  is on the verge of leaving  $L$  as shown in Fig. 10c. At that instant,  $\bar{u}$  is suddenly changed to  $\bar{u} = [\frac{3}{4}, \frac{1}{4}]^T$ , which pushes  $u^*$  back into the feasible region, where it remains for the duration of the motion.

**Example 4, in Fig. 11:** This is a suboptimal trajectory. The input is initially at  $\bar{u} = [\frac{1}{4}, \frac{3}{4}]^T$ , which again causes  $u^*$  to move down the curve. Since  $\bar{u} \neq u^*$ ,  $u^*$  moves away from  $\bar{u}$  (see Fig. 11b) according to (15). The necessary condition of optimality is satisfied through  $t = 1$ , after which  $u^*$  leaves the feasible region (Fig. 11c). The trajectory is confirmed to be suboptimal immediately after  $u^*$  leaves  $L$  and enters  $M_1$ —the suboptimality is proven even before  $\bar{u}$  violates the necessary condition. Once the point  $u^*$  leaves  $U$ , then eventually the input  $\bar{u}$  must become sub-optimal. At  $t = \frac{4}{3}$ , actuator 2 reaches the origin and the dimension is reduced by one, as in Fig. 11d. Due to the constraint  $u_1 < d_1$ , it is impossible for  $\bar{u}$  to continue satisfying the optimality condition. Indeed,  $x$  reaches the origin with  $T = 2\frac{2}{9}$ , which is 11 percent longer than the optimal cases. This illustrates the potentially counter-intuitive result that an operator commanding inputs that at all times yield the maximum speed of an actuator will not necessarily yield a time-optimal trajectory.

## IX. CONCLUSIONS

This paper gave the optimality conditions on the input  $u(t)$  for completing a motion primitive with a displacement  $x(t)$  with multi- and single-actuator constraints on the allowable input. The solution was derived by assuming a quasi-dynamic

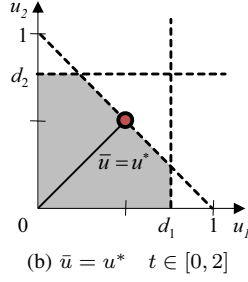
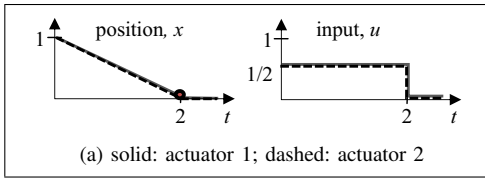


Fig. 9: Example 2: An optimal trajectory with  $\bar{u} = u^*$ .

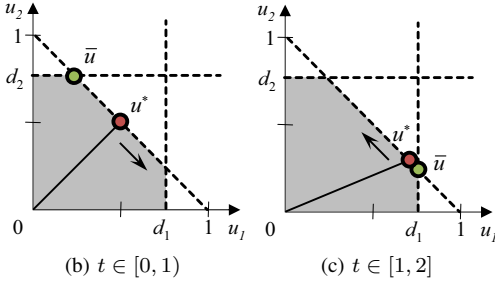
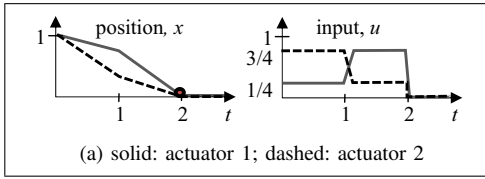


Fig. 10: Example 3: An optimal trajectory with  $\bar{u} \neq u^*$ . (a): The actuator position and velocity command. (b):  $\bar{u} = [\frac{1}{4}, \frac{3}{4}]^T$ . (c):  $\bar{u} = [\frac{3}{4}, \frac{1}{4}]^T$ .

model for the actuator dynamics. This assumption produced a solution amenable for real-time computation on low-end industrial controllers typical in industry. The optimal solution is almost never unique, but optimality can be tested by considering a special point  $u^*$ . The location of  $u^*$  in the input plane provides a convenient test for optimality, and can discriminate an input  $u(t)$  as sub-optimal even before the input violates a necessary condition for optimality, as demonstrated in the four examples presented.

There are some clear caveats to this optimization approach. First, the relative distance  $x(t)$  was assumed to be precisely known. In reality,  $x(t)$  is estimated online and is subject to error. This error manifests as error in  $u^*$ . The effects of this error on task completion time should be studied. Second, the optimization occurs relative to each motion primitive; whether sequential optimization of a trajectory's constituent

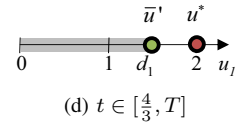
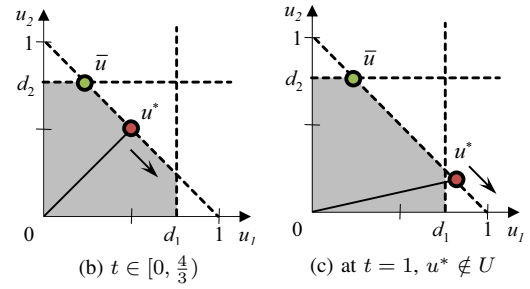
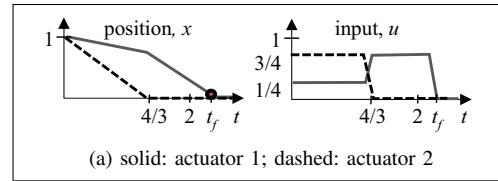


Fig. 11: Example 4: A suboptimal trajectory. (a): The actuator position and velocity command. (b):  $\bar{u} = [\frac{1}{4}, \frac{3}{4}]^T$  (d):  $\bar{u} = [\frac{3}{4}, 0]^T$

primitives leads to a lower overall task cost remains to be shown. Third, the controls engineer must weigh whether minimizing task time is appropriate for a given application, especially since the energetic expense of a manipulator trajectory tends to increase with the speed.

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